wire no.   17   16   15   14   13   12   11   10   9   8   7   6   5   4   3   2   1   0   *(101)*

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ \alpha^{15} & \alpha^{14} & \alpha^{13} & \alpha^{12} & \alpha^{11} & \alpha^{10} & \alpha^{9} & \alpha^{8} & \alpha^{7} & \alpha^{6} & \alpha^{5} & \alpha^{4} & \alpha^{3} & \alpha^{2} & \alpha & 1 & 1 & 0 \end{bmatrix}$$ *(103)*

FIG. 1 (PRIOR ART)

wire no. $\cdots$ 4   3   2   1   0   *(401)*   *(400)*

$$H = \begin{bmatrix} \cdots & 1 & 1 & 1 & 0 & 1 \\ \cdots & \alpha^{B+2} & \alpha^{B+1} & \alpha^{B} & 1 & 0 \end{bmatrix}$$ *(403)   (405)*

FIG. 4

FIG. 2
(PRIOR ART)

FIG. 3

500 *ENCODING*

501

CALCULATE HORIZONTAL PARITY, HP[i] AS:

$$HP[i] = \overset{18}{\underset{wire=2}{\oplus}} b[wire][i] \text{, i.e., } HP[0] = b[2][0] \oplus b[3][0] \oplus \ldots \oplus b[18][0]$$

503

CALCULATE EXTENDED PARITY, EP[7:0], WHERE EP IS A DEGREE 7 POLYNOMIAL

$$EP = \sum_{x=2}^{18} \sum_{y=0}^{7} b[x][y] \, \alpha^{(x+y-2)}$$

e.g., EP[0] IS A COEFFICIENT OF α^0, EP[1] IS A COEFFICIENT OF α^1,

...

AND EP[7] IS A COEFFICIENT OF α^7

505

SEND CONTENTS OF HP ALONG WIRE 0

(HP[0] IN SYMBOL 0, HP[1] IN SYMBOL 1,..., HP[7] IN SYMBOL 7)

507

SEND CONTENTS OF EP ALONG WIRE 1

(EP[0] IN SYMBOL 0, EP[1] IN SYMBOL 1, ..., EP[7] IN SYMBOL 7

509

SEND CONTENTS OF INFORMATION BIT b[i][j] ALONG WIRE i,
FOR i=2 TO 18 AND j=0..7
b[i][0] IN SYMBOL 0, b[i][1] IN SYMBOL 1, ..., b[i][7] IN SYMBOL 7

FIG. 5
(PRIOR ART)

600 DECODING

601

CALCULATE SYND 0, REPRESENT 0 AS {e7, e6, ..., e0}, AN 8 BIT QUANTITY, WHERE,

$$e[i] = \left(\bigoplus_{x=2}^{18} b[x][i] \right) \oplus b[0][i]$$

603

CALCULATE SYND 1, A POLYNOMIAL OF DEGREE 7

$$synd\ 1 = \left(\sum_{i=0}^{7} b[1][i]\alpha^i \right) + \sum_{x=2}^{18} \sum_{y=0}^{7} b[x][y]\ \alpha^{x+y-2}$$

605

synd 0 = 0 ?

YES   NO

607

synd 1 = 0 ?

609

synd 1 = 0 ?   NO → (B)

YES   611   NO   613   YES   615

| 1. NO ERROR
2. USE THE BITS [b18:2][7:0] AS DATA | 1. ERROR IN WIRE 1
2. USE THE BITS b[18:2][7:0] UNMODIFIED AS DATA | 1. ERROR IN WIRE 0
2. USE THE BITS b[18:2][7:0] UNMODIFIED AS DATA |

FIG. 6A *(PRIOR ART)*

B

621

ERR_WIRE = 2

623

CALCULATE: SYND1_IF_ERROR_IN_ERR_WIRE =

$\alpha^{(err\_wire - 2)}$ (e7α^7 + e6α^6 + e5α^5 + ... + e1α + e0)

625

synd1_if_error_in_wire = synd1
?

YES

NO

631

err_wire = err_wire + 1

NO

629

err_wire >
18?

627

0. BITS IN ERROR {e7, e6, ..., e0}

1. ERROR IN WIRE err_wire

2. FLIP THE BITS b[err_wire][i]
BY e[i] for i=0..7

3. USE THE OTHER BITS IN
OTHER WIRES b[x][i] FOR I=0..7,
X=2..18 AND X ≠ err_wire
WITHOUT MODIFICATION AS
DATA

YES

633

1. MULTI-WIRE ERROR

2. CANNOT USE THE
DATA

FIG. 6B (PRIOR ART)

700 ENCODING

701

CALCULATE HORIZONTAL PARITY, HP[i] FOR i=0..7

$$HP[i] = \bigoplus_{x=0}^{17} b[x][i] \text{, i.e., } HP[0] = b[0][0] \oplus b[1][0] \oplus \ldots \oplus b[17][0],$$

where info bits are b[17][8], and B=degree of primitive polynomial +1

703

CALCULATE EXTENDED PARITY, EP[7:0], WHERE B IS THE DEGREE OF
PRIMITIVE POLYNOMIAL + 1 = NUMBER OF BITS IN synd 0 OR synd 1

$$EP = \sum_{x=0}^{16} \sum_{y=0}^{7} b[x][y] \, \alpha^{(x+y+B)}$$

e.g.,   SINCE THE POLYNOMIAL IS OF DEGREE 7,
EP[i] IS THE COEFFICIENT OF α^{i+B}

707 CALCULATE OVERALL PARITY (OP), WHERE

$$OP = HP[0] \oplus HP[1] \oplus \ldots \oplus HP[7] \oplus EP[0] \oplus EP[1] \oplus \ldots \oplus EP[7]$$

709

SEND CHECK BITS (EP[7:0], HP[7:0], OP) IN SYMBOL 0 WITH EP[7] IN
WIRE 16. HP[0] IN WIRE 1 AND OP IN WIRE 0. NOTE ANY SYMBOL
MAY BE USED, IN ANY ORDER.

THE REST OF THE BITS ARE PUT INTO SYMBOLS 1THOUGH 8;
SYMBOL [i] CARRIES b[16..0][i]

FIG. 7

800 DECODING

802

CHECK IF THE CHECK BITS HAVE AN ERROR OR NOT, WHERE

$$\text{error\_checkbits} = OP\ (b[0][0])\ \oplus\ \left(\ \overset{7}{\underset{i=0}{\oplus}}\ HP[i]\ \right) \oplus \left(\ \overset{7}{\underset{i=0}{\oplus}} EP[i]\right),$$

AND WHERE HP[i] = b[i+1][0] and EP[i] = b[i+9]

803

D2
error checkbits = 0? — NO → C

YES (No error in check bits)

804

CALCULATE synd 0 AS AN 8-BIT QUANTITY {e7, e6, ..., e0}

$$e[i] = \left(\ \overset{17}{\underset{x=0}{\oplus}}\ b[x][1]\ \right) \oplus HP[],\ \text{WHERE}\ HP[i] = b[i+1][8]$$

CALCULATE synd 1 AS A DEGREE 7 POLYNOMIAL, WHERE

$$\text{synd 1} = \sum_{i=0}^{7} EP[i]\alpha^{i} + \sum_{x=0}^{16} \sum_{y=0}^{7} b[x][y]\ \alpha^{x+y+B}\ ,$$

WHERE EP[i] $]\alpha^{i}$ = b[i+9][0]

806

808

synd0 = 0 ?
YES ← → NO

810
synd1 = 0 ? — NO →

YES

814
synd1 = 0 ? — NO → A
YES ←

812

1. NO ERROR

2. USE BITS
b[16:0][8:1] AS
INPUT DATA

816

1. MULTI-WIRE ERROR (SINCE
OVERALL PARITY CALCULATION
INDICATED TAT CHECKBITS
WERE ERROR-FREE. ANY WIRE
ERROR IN DATA BITS WILL
CORRUPT BOTH SYNDROMES)

2. DO NOT USE DATA

FIG. 8A

A

818

err_wire = 0

820

CALCULATE THE SYNDROME ASSUMING ERROR HAPPENED
IN WIRE "err_wire"

$synd1\_if\_error\_in\_'err\_wire' = \propto^{(err\_wire + B)} (e7\propto^7 + e6\propto^6 + ... + e0)$

822

NO

$synd1 = synd1\_if\_error\_in\_'err\_wire'$
?

YES

824

826

err_wire ≥16
?

YES

NO

0. BITS IN ERROR: $\{e^7, e^6, .., e0\}$
1. ERROR IN WIRE: err_wire
2. FLIP THE BITS b[err_wire][i] BY e[i] FOR
i=0..7 AND USE AS DATA

3. USE OTHER BITS IN OTHER WIRES b[x][i]
FOR i=0..7, x=0..16 AND x ≠ err_wire
WITHOUT MODIFICATION AS DATA

828

1. MULTI-WIRE ERROR
2. DO NOT USE DATA

830

err_wire = err_wire + 1

FIG. 8B

C ⟶ err_wire = 0 *832*

834 — PERFORM STEPS 804, 806, 808, 810, 814, 812, 816

836 — PERFORM STEPS 820, 822, 824

838 — IF 'NO' IN STEP D12 ? — NO, not possible

YES

840 — err_wire = 1

D22 ⟶ HP'[i] = HP[i]
EP'[i] = EP[i],
for i=0..7   *842*

844 — err_wire < 9 ? — YES ⟶ *845* HP'[err_wire −1] = ~HP'[err_wire −1]

NO

846 — EP'[err_wire] = ~EP'[err_wire − 9]

Here, HP' and EP' reflect the HP and EP after flipping the required bit, assuming an error in err-wire

848 — CALCULATE synd 0 AS AN 8-BIT QUANITITY {e7, e6, ..., e0}

$$e[i] = \bigoplus_{x=2}^{8} b[x][i+1] \oplus HP'[i] \quad synd0 = \{e7,..,e0\}$$

D

FIG. 8C

(D)

850

CALCULATE SYND 1 AS A DEGREE 7 POLYNOMIAL,

$$\text{Synd } 1 = \sum_{i=0}^{7} EP'[i]\, \alpha^i + \sum_{x=0}^{16} \sum_{y=1}^{8} b[x][y]\, \alpha^{x+y-1+B}$$

852

YES ← synd 0 = 0? → NO

854

synd 1 = 0? → NO

YES

856

synd 1 = 0?

NO

858

synd 1 = $\alpha^{(err\_wire + B)}$ synd 0 ?

YES ↓

862

1. ERROR IN SYMBOL 8, WIRE NUMBER err_wire

2. USE BITS b[16..0][7..0] AS INPUT DATA

NO

866

1. ERROR IN SYMBOL 8 AS WELL AS SYMBOLS WHERE {e0, e1, .., e7} ARE NON-ZERO (i.e., '1'). IF e_i = 1 THEN SYMBOL i HAS AN ERROR. WIRE LOCATION IS 'err_wire'

2. FLIP THE BITS IN b[err_wire][i] BY e[i] FOR i=0..7

3. USE OTHER BITS IN OTHER WIRES b[x][i] FOR i=0..7, x=0..16 AND x≠err_wire WITHOUT MODIFICATION AS DATA.

860

YES ← err_wire ≥ 16 ?

MULTI-WIRE ERROR DO NOT USE DATA

864

NO

err_wire = err_wire + 1

868

(D22)

FIG. 8D